1 **A**

$$\begin{aligned} P^2 &= 4I \\ \frac{1}{4}PP &= I \\ \left(\frac{1}{4}P\right)P &= I. \end{aligned}$$

Therefore,

$$P^{-1} = \frac{1}{4}P.$$

2 **B** $RS = [5(0) + (3)(-1) + (1)(2)] = [-1]$

3 **E** $\det A = (9)(5) - (8)(-11) = 133$

4 **A** The product of an 1×3 matrix by a 3×1 matrix will be a 1×1 matrix.

5 **B**

$$\begin{aligned} AX + B &= C \\ AX &= C - B \\ X &= A^{-1}C - B \\ &= \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 1 \\ 4 & 0 \end{bmatrix} \end{aligned}$$

6 **C** Since $PQR = \begin{bmatrix} 7 & 0 \\ 0 & 56 \end{bmatrix}$, there are 2 zero entries.

7 **A**

$$\begin{aligned} X^{-1} &= \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \frac{1}{3(-2) - (5)(-1)}\begin{bmatrix} -2 & -5 \\ 1 & 3 \end{bmatrix} \\ &= \frac{1}{-1}\begin{bmatrix} -2 & -5 \\ 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 5 \\ -1 & -3 \end{bmatrix} \end{aligned}$$

8 **B** $\det A = ad - bc = (4)(4) - (6)(2) = 4$

9 **D**

$$\begin{aligned} S^{-1} &= \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \frac{1}{5(2) - (7)(2)}\begin{bmatrix} 2 & -7 \\ -2 & 5 \end{bmatrix} \\ &= \frac{1}{4}\begin{bmatrix} -2 & 7 \\ 2 & -5 \end{bmatrix} \end{aligned}$$

10 **D** A reflection in the line $y = x$ is given by the rule $(x, y) \to (y, x)$. Therefore, $(5, -2) \to (-2, 5)$.

11 **C** A reflection in the line $y = -x$ is given by the rule $(x, y) \to (-y, -x)$. Therefore, $(2, -6) \to (6, -2)$.

12 **B** The point $(5, -4)$ is translated to the point $(7, -7)$. Its distance to the line $y = 1$ is 8. Therefore, it will deflect to a point 8 units on the other side of the line $y = 1$. That is, to the point $(7, 9)$.

13 **A** We can think of this as a translation of (a, b) to the line $x = m$ by translating the point by $m - a$ units in the x-direction, then a further $m - a$ units in the x-direction. The x-coordinate will then be

$$a + (m - a) + (m - a) = 2m - a.$$

the y-coordinate is unchanged.

14 **E** If the line $x+y=4$ is dilated from the y-axis by a factor of $\frac{1}{2}$ its new equation will be $y+2x=4$. We reflect its intercepts $(0,4)$ and $(2,0)$ in the line $x=4$ to the points $(8,4)$ and $(6,0)$ respectively. The straight line through these two points has equation $y=2x-12$.

15 **B** The required transformation is

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+3 \\ -(y+2) \end{bmatrix}.$$

Therefore, $x'=x+3$ and $y'=-y-2$. Solving for x and y gives,

$x=x'-3 \text{ and } y=-y'-2$

so that $y=x^2$ becomes $-y'-2=(x'-3)^2$. Solving for y' gives,

$y'=-(x'-3)^2-2.$

Deleting the dash symbols leaves $y=-(x-3)^2-2$, which corresponds to item B.

16 **C** The required transformation is

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{x}{3} \\ 2y \end{bmatrix}.$$

Therefore, $x'=\frac{x}{3}$ and $y'=2y$. Solving for x and y gives,

$$x=3x' \text{ and } y=\frac{y'}{2}$$

so that $y=2^x$ becomes $\frac{y'}{2}=2^{3x'}$. Solving for y' gives,

$$y'=2\times 2^{3x'}.$$

Deleting the dash symbols leaves $y=2\times 2^{3x}$.

17 **D** A reflection in the line $x=2$ is given by the rule

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4-x \\ y \end{bmatrix}.$$

If we then perform the translation we obtain the transformation,

$$\begin{aligned} \begin{bmatrix} x' \\ y' \end{bmatrix} &= \begin{bmatrix} 4-x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 6-x \\ y+3 \end{bmatrix} \end{aligned}$$

18 **D** The matrix of the transformation is

$$B = \begin{bmatrix} 4 & 3 \\ 4 & 5 \end{bmatrix}.$$

The inverse transformation will have matrix

$$\begin{aligned} B^{-1} &= \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \frac{1}{4(4)-(5)(3)}\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}, \end{aligned}$$

which corresponds to item D

19 **E** The matrix of this transformation will be

$$\begin{bmatrix} 0 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}.$$

The first column corresponds to the image of $(1, 0)$ and the second to the image of $(0, 1)$. Therefore,

$(1, 0) \to (0, 2)$ and $(0, 1) \to (-1, 1)$.

Only item E contains both of these points.

20 **D** A rotation by $35°$ clockwise then $15°$ anticlockwise is a rotation by $20°$ clockwise. The has transformation matrix,

$$\begin{bmatrix} \cos(-20)° & -\sin(-20)° \\ \sin(-20)° & \cos(-20)° \end{bmatrix} = \begin{bmatrix} \cos 20° & \sin 20° \\ -\sin 20° & \cos 20° \end{bmatrix}$$

21 **B** An anticlockwise rotation by angle θ is given by the matrix,

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

This cannot be a reflection because of the location of the negative entry.

22 **B** The area of the original square is 1. Therefore the image will have area,

$$\begin{aligned} \text{area of image} &= |\det B| \times \text{original area} \\ &= |2(5) - (3)(4)| \times 1 \\ &= 2. \end{aligned}$$

23 **B** Since

$$|\boldsymbol{a}| = \sqrt{3^2 + 4^2} = 5,$$

the unit vector will be

$$\frac{\boldsymbol{a}}{|\boldsymbol{a}|} = \frac{1}{5}(3\boldsymbol{i} + 4\boldsymbol{j}).$$

24 **D**
$$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\ &= (3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}) - (2\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}) \\ &= \boldsymbol{i} + 8\boldsymbol{j} \end{aligned}$$

25 **B**
$$\begin{aligned} \boldsymbol{a} - \boldsymbol{b} &= (2\boldsymbol{i} + 4\boldsymbol{j}) - (3\boldsymbol{i} - 2\boldsymbol{j}) \\ &= -\boldsymbol{i} + 6\boldsymbol{j} \end{aligned}$$

26 **A** $|\boldsymbol{a}| = \sqrt{2^2 + (-1)^2 + (4)^2} = \sqrt{21}$

27 **B**
$$\begin{aligned} \overrightarrow{AD} &= \overrightarrow{AB} + \overrightarrow{BO} + \overrightarrow{OC} + \overrightarrow{CD} \\ &= \boldsymbol{c} + -\boldsymbol{b} + \boldsymbol{c} + -\boldsymbol{b} \\ &= 2\boldsymbol{c} - 2\boldsymbol{b} \\ &= 2(\boldsymbol{c} - \boldsymbol{b}) \end{aligned}$$

28 **D**
$$\begin{aligned} 2\boldsymbol{r} - \boldsymbol{s} &= 2(2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}) - (-\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}) \\ &= 5\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k} \end{aligned}$$

29 **A**

30 **B** Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are parallel if

$$\begin{aligned} \boldsymbol{u} &= c\boldsymbol{v} \\ \boldsymbol{i} + a\boldsymbol{j} - 5\boldsymbol{k} &= cb\boldsymbol{i} - 3c\boldsymbol{j} + 6c\boldsymbol{k} \end{aligned}$$

Equating coefficients gives $c = -\frac{5}{6}$ and

$$a = -3c = -3 \times -\frac{5}{6} = \frac{5}{2},$$
$$b = 1 \div c = -\frac{6}{5}.$$

31 **C**

$$\boldsymbol{x} = s\boldsymbol{a} + t\boldsymbol{b}$$
$$\boldsymbol{i} + 5\boldsymbol{j} = 3s\boldsymbol{i} + 4s\boldsymbol{j} + 2t\boldsymbol{i} - t\boldsymbol{j}$$
$$\boldsymbol{i} + 5\boldsymbol{j} = (3s + 2t)\boldsymbol{i} + (4s - t)\boldsymbol{j}$$

Therefore, $3s + 2t = 1$ and $4s - t = 5$. Solving these simultaneous equations gives,

$$s = 1, t = -1.$$

32 **B**

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OC} + \overrightarrow{CB} \\ &= -\boldsymbol{a} + \boldsymbol{c} + \frac{1}{3}\boldsymbol{a} \\ &= \boldsymbol{c} - \frac{2}{3}\boldsymbol{a}\end{aligned}$$

33 **B**

$$\begin{aligned}\boldsymbol{c} &= \overrightarrow{OC} \\ &= \overrightarrow{OB} + \overrightarrow{BA} + \overrightarrow{AC} \\ &= \boldsymbol{b} + (\boldsymbol{a} - \boldsymbol{b}) + 2(\boldsymbol{a} - \boldsymbol{b}) \\ &= \boldsymbol{b} + 3(\boldsymbol{a} - \boldsymbol{b}) \\ &= 3\boldsymbol{a} - 2\boldsymbol{b}\end{aligned}$$